Bayesian Alpha Estimation via Fama–French + Momentum Model

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Abstract

This paper proposes a Bayesian long-short equity strategy built on the Fama–French five-factor model with an added momentum component. Using Bayesian linear regression, the model estimates asset-level alpha and selects positions based on posterior credible intervals, forming a market-neutral portfolio of high-confidence signals.

The strategy is tested on a diversified universe of U.S.-listed ETFs and large-cap stocks across sectors and styles, with a 52-week rolling window and bi-monthly rebalancing. It consistently outperforms SPY, equal-weight, and random strategies in both return and risk-adjusted metrics. Posterior predictive checks and prior-posterior comparisons confirm the model learns meaningful signals from noisy data. A sector-level case study on biotech equities (XBI) further demonstrates the approach's generalizability. Results highlight the strengths of Bayesian inference in filtering alpha and improving systematic investment outcomes.

1. Introduction

Generating persistent alpha remains a key challenge in quantitative finance. While models like Fama–French capture broad return variation, they often miss idiosyncratic performance. Frequentist alpha estimates tend to be noisy and unstable, especially in high-dimensional settings.

This study presents a Bayesian alternative that incorporates prior beliefs and quantifies uncertainty in parameter estimation. By applying Bayesian linear regression and filtering based on posterior credible intervals, the method avoids spurious signals and emphasizes statistical robustness. The strategy is tested across a diversified universe of U.S. equity ETFs and stocks, producing a market-neutral long-short portfolio with controlled risk. A sector-specific case study in biotech further illustrates the model's flexibility. Overall, this Bayesian framework offers a more disciplined and interpretable approach to alpha selection than traditional signal-based methods.

2. Methodology

2.1 Data Sources and Universe Construction

The asset universe consists of 30 U.S.-listed ETFs and large-cap stocks spanning sectors (e.g., healthcare, industrials), investment styles (e.g., momentum, value, quality), and size tiers. Mega-cap tech names are excluded to avoid index concentration. Weekly total returns are computed from adjusted Friday closing prices between January 2014 and January 2024. Securities with full-sample volatility under 1% are removed.

Factor data, covering the Fama–French five factors and a momentum factor, is sourced from Kenneth French's database and resampled to weekly frequency. Weekly excess returns are computed using the one-week Treasury bill yield.

2.2 Bayesian Regression Model

We estimate each asset's alpha using Bayesian linear regression based on the Fama–French five-factor model with an added momentum factor. The model takes the form:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,M}\left(R_{M,t} - R_{f,t}\right) + \beta_{i,S} \cdot SMB_t + \beta_{i,V} \cdot HML_t + \beta_{i,P} \cdot RMW_t$$
$$+\beta_{i,I} \cdot CMA_t + \beta_{i,MOM} \cdot MOM_t + \varepsilon_{i,t}$$

Here, $R_{i,t}$ is the total return of asset i, $R_{f,t}$ is the risk-free rate, and the factor returns represent market excess return (MKT), size (SMB), value (HML), profitability (RMW), investment (CMA), and momentum (MOM). The intercept α_i captures asset-specific abnormal return unexplained by the factors.

We impose a conjugate Normal-Inverse-Gamma prior:

- Prior on coefficients**:** $(\alpha_i, \beta_i) \sim \mathcal{N}(\mu_0, V_0)$, with tighter variance on α_i
- Prior on variance**:** $\sigma_i^2 \sim \text{Inverse-Gamma}\left(\frac{\nu_0}{2}, \frac{\nu_0 s_0}{2}\right)$
- Posterior covariance: $V_N = \left(V_0^{-1} + \frac{1}{\sigma^2} X^\top X\right)^{-1}$
- Posterior mean: $\mu_N = V_N\left(V_0^{-1}\mu_0 + \frac{1}{\sigma^2} X^\top y\right)$

This setup enables closed-form posterior inference. The marginal posterior of α_i follows a Student-t distribution, from which we extract both the mean and the 95% credible interval for signal selection.

2.3 Signal Selection and Portfolio Construction

At each rebalance point (every 8 weeks), we apply a 52-week rolling estimation window. Assets with 95% posterior HDIs for α_i entirely above or below zero are selected as long or short candidates. Up to 3 per side are chosen by posterior mean magnitude. Positions are equal-weighted and market-neutral; unallocated capital remains in cash. A 0.20% transaction cost per trade is applied. This confidence-driven construction enables dynamic exposure scaling and embedded risk control.

3. Backtesting and Results

3.1 Strategy Comparison

	SPY	StaticEqualWeight	RandomLongShort	BayesianStrategy
Annual Return	-0.0785	-0.0366	-0.0863	0.483
Volatility	0.2183	0.2136	0.0979	0.3054
Sharpe Ratio	-0.3596	-0.1714	-0.8811	1.5818
Max Drawdown	0.1353	0.1167	0.061	0.1677

Figure 1: Performance Metrics Across Strategies

The Bayesian strategy achieved an annualized return of 48.3% with a Sharpe ratio of 1.58, significantly outperforming all benchmark strategies. In contrast, the SPY benchmark recorded an annual return of –7.85% and a Sharpe ratio of –0.36, while the static equal-weight portfolio and the random long-short strategy posted –3.66% and –8.63% returns, respectively.

While the Bayesian strategy exhibited the highest annualized volatility at 30.5%, this reflects its dynamic long-short positioning. Despite the elevated risk, its risk-adjusted performance remained superior, as evidenced by the substantially higher Sharpe ratio. The maximum drawdown of the Bayesian strategy was 16.8%, which, although higher than SPY's (13.5%) and the static portfolio's (11.7%), remained within a moderate range.

Overall, the Bayesian strategy demonstrated strong outperformance in both absolute and risk-adjusted terms, validating its effectiveness in dynamically allocating across assets.

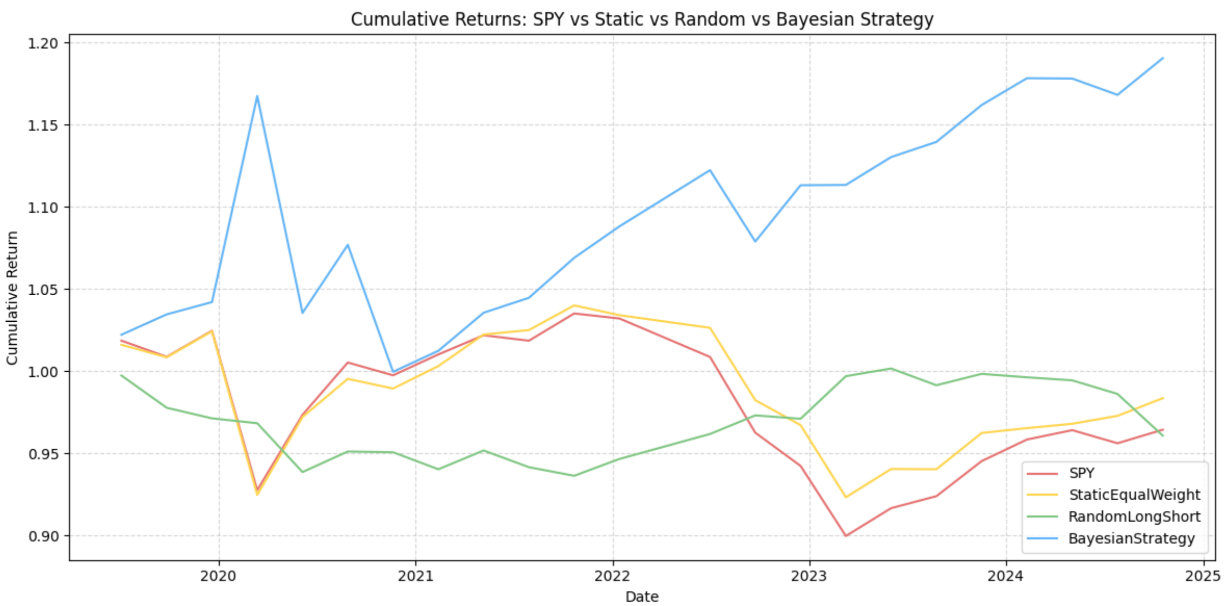

Figure 2: Cumulative Returns of Bayesian Strategy and Benchmarks

3.2 Alpha Learning Diagnostics

Posterior predictive checks confirm that the Bayesian model's simulated returns closely align with actual returns within the 95% highest density intervals (HDIs), indicating strong calibration. This suggests the model effectively captures the distributional characteristics of observed asset returns. In addition, prior–posterior comparisons reveal a clear shift in the distribution of estimated alphas away from the neutral prior centered at zero. This reflects the model's ability to learn from data: while the prior assumes no persistent alpha, the posterior concentrates around positive values, implying the model has identified systematic excess returns not explained by common risk factors.

The model illustrates a core feature of Bayesian inference: it begins with conservative assumptions ($\alpha \approx 0$) but updates beliefs as evidence accumulates. Over time, the posterior increasingly reflects assets with consistent outperformance, indicating the presence of exploitable alpha.

With a mean rolling R^2 of 0.77 across estimation windows, the model demonstrates strong in-sample explanatory power. These diagnostics collectively highlight that Bayesian inference offers a principled, data-driven framework for uncovering meaningful signals in noisy financial environments.

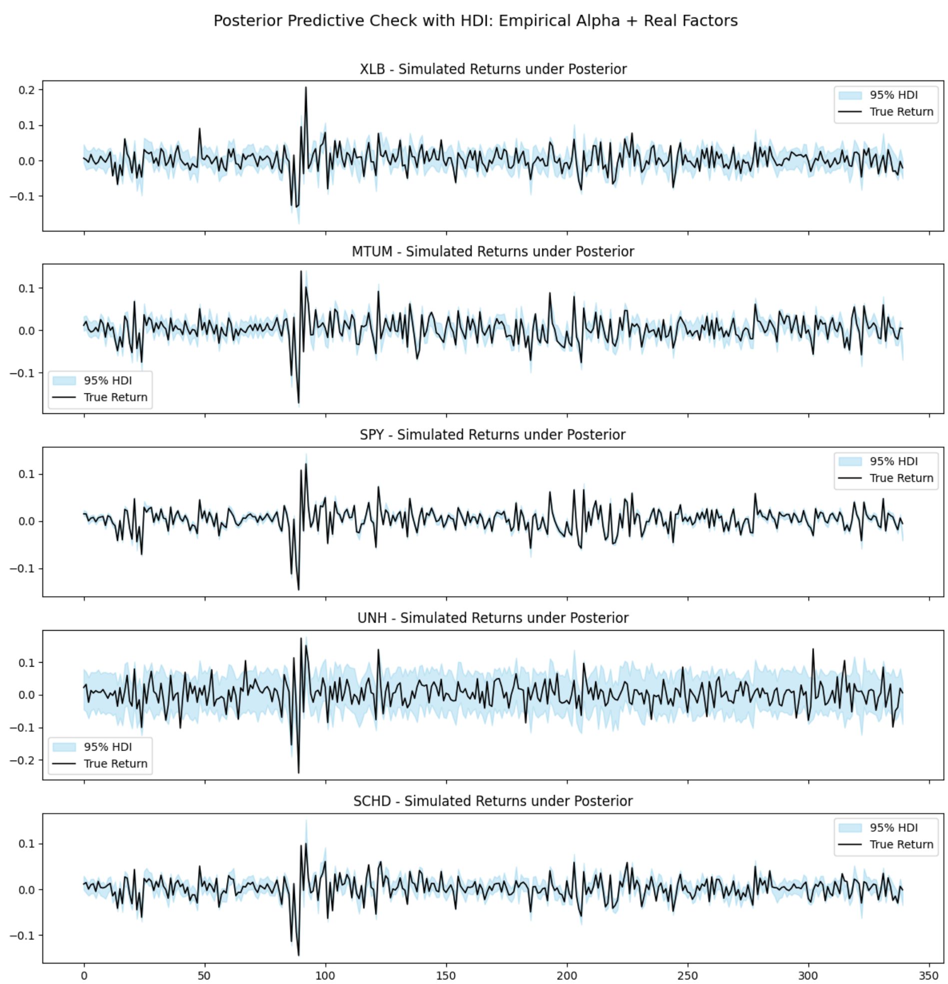

Figure 3: Posterior Predictive Checks with 95% HDI for Selected Assets

4. Discussion

The Bayesian long-short strategy delivers strong absolute and risk-adjusted performance, achieving an annual return of 48.3% and a Sharpe ratio of 1.58. This clearly outpaces traditional benchmarks such as SPY, the static equal-weight portfolio, and the random long-short strategy. As shown in Figure 1, the Bayesian strategy exhibits a steadily rising return path, highlighting the value of disciplined alpha selection based on posterior inference.

The use of highest density intervals (HDIs) to filter significant alphas helps mitigate overfitting by excluding spurious signals. Combined with posterior predictive checks and a high rolling R^2, this indicates that the model captures persistent and interpretable return patterns.

However, limitations remain. The framework is computationally intensive, especially with large asset universes or more frequent rebalancing. Its reliance on static priors may also constrain adaptability in fast-changing markets. Moreover, the strategy tends to abstain from trading when posterior alpha confidence is low, which may lead to missed opportunities during turbulent regimes.

To improve adaptability, future research could explore hierarchical or time-varying priors, enabling cross-asset learning and dynamic signal strength. Integrating posterior outputs into Bayesian portfolio optimization, or applying the model to global markets or alternative assets, may further extend its practical relevance.

5. Conclusion

This paper presents a Bayesian long-short equity strategy built on the Fama–French five-factor model with an added momentum component. By using Bayesian linear regression with conjugate priors and filtering based on posterior credible intervals, the framework identifies statistically meaningful alpha signals and constructs a market-neutral portfolio with high confidence.

Backtesting results show that the Bayesian strategy consistently outperforms traditional benchmarks in both total return and risk-adjusted terms. Despite its active long-short positioning, the strategy maintains controlled volatility and drawdowns, and achieves a mean rolling R^2 of

0.77, indicating strong explanatory power. Posterior predictive checks and prior–posterior comparisons confirm that the model effectively learns from data while avoiding overfitting.

These findings underscore the value of Bayesian methods in enhancing the robustness, selectivity, and interpretability of quantitative investment models. Future extensions could incorporate dynamic priors, hierarchical structures, or multi-asset universes to further broaden the framework's reach and adaptability.

6. References

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