Physics-Informed Neural Networks for Futures Pricing and Signal-Generation Under Stochastic Volatility

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1 Abstract

Traditional mathematical models provide a generalized approach for calculating futures prices that may not always capture hidden relationships between the asset's spot price and futures price. For instance, the Cost-of-Carry model neglects any potential effects of volatility on the futures price, and the Black-Scholes model disregards the randomness involved in the asset's volatility. Other models, such as the Heston model, succeed in capturing the stochastic nature of volatility, but they tend to be computationally intensive. Although mathematical models are important for precisely calculating futures prices, it would be largely beneficial to explore alternative approaches that could provide increased efficiency. As such, this project aims to identify the benefits of using neural partial differential equations (PDEs) as alternatives to pure numerical methods. This approach leverages Physics-Informed Neural Networks (PINNs) to solve the Feynman-Kac PDE using the Heston model. The prices produced by the PINN will then be compared against prices obtained by brokers to analyze potential pricing discrepancies and generate signals for future price movements.

2 Introduction

This project aims to evaluate the performance of Physics-Informed Neural Networks for calculating futures prices across asset classes. This research aims to show that PINNs can be used as a reliable tool for efficiently calculating futures prices for fixed income, equity, and commodity markets. By leveraging this tool, AlgoGators can seamlessly develop an internal futures pricing model that can help them exploit investment opportunities from potential contract mispricing.

3 Methodology

The project must begin by determining the ideal combinations of spot prices and futures prices for training the model. This process involved examining the correlations between different tickers' daily returns and using these as proxies for how well they will serve for measuring the contract's spot price. To do so, I extracted data from Yahoo Finance for each futures contract's corresponding ETFs and indices. After examining these relationships, I selected the following assets for further examination:

Asset Type	Asset	Futures Ticker	Spot Ticker	Correlation
Fixed Income	10-Year T-Note Futures	ZN=F	IEF	0.92
Fixed Income	30-Year T-Bond Futures	ZB=F	TLT	0.88
Equities	Russell 2000	RTY=F	IWM	0.98
Equities	Dow Jones	YM=F	DIA	0.97
Commodities	Gold	GC=F	GLD	0.90

Before moving forward with the neural network, it is important to understand the mathematical foundations that underly the physics-informed loss function. My research focuses on using the Feynman-Kac PDE based on the stochastic differential equation (SDE) from the Heston model. The Heston model SDE in a risk neutral environment is defined as follows:

$$dS_t = rS_t dt + S_t\sqrt{v_t}\Sigma_S^{\mathrm{T}} dW_t$$
$$dv_t = \kappa(\gamma - v_t)dt + \sigma\sqrt{v_t}\Sigma_v^{\mathrm{T}} dW_t,$$

where S_t is the spot price of the asset at time t, v_t is the variance of the asset, r is the risk-free rate, κ is the rate of mean-reversion, γ is the long-run variance, σ is the volatility, Σ is the diffusion, and W_t is a vector of two independent Brownian motions. Based on this SDE, we can obtain the following Feynman-Kac formula:

$$0 = \partial_t V - rV + \mu_y^{\mathrm{T}} \nabla_y V + \tfrac{1}{2} trace\left(\Sigma_y \Sigma_y^{\mathrm{T}} H_y(V)\right),$$

which can be rewritten as:

$$0 = \frac{\partial V}{\partial t} - rV + \left(\mu_S \frac{\partial V}{\partial S_t} + \mu_v \frac{\partial V}{\partial v_t}\right) + \frac{v}{2}\left(S^2 \frac{\partial^2 V}{\partial {S_t}^2} + 2\rho\sigma S \frac{\partial^2 V}{\partial v \partial S_t} + \sigma^2 \frac{\partial^2 V}{\partial {v_t}^2}\right),$$

where V is the value function for futures price, μ_S is the drift of the spot price, μ_v is the drift of variance, and ρ is the correlation between stock price and variance. The formula consists of four main terms: the derivative of the value function with respect to time, the product of the risk-free rate and value, the drift term, and the trace term. In the case of our PINN, the value will be defined as the model's output for the futures price calculation. The value function must follow the terminal condition

$$V(T, y_t) = S_t,$$

meaning that the futures price should converge to the spot price at maturity.

To train the model, I collected data for each of the parameters involved in the PDE. I used Yahoo Finance to get historic open, high, low, close, and volume data for the futures contract and underlying asset. I used the FRED api to obtain the three-month treasury rate, which would then be used as the risk-free rate. Then, I used NumPy and Pandas to calculate the drift of the spot price μ_S, drift of variance μ_v, volatility σ, long-run variance γ, rate of mean-reversion κ, correlation between stock price and variance ρ, and the time t. All of the above data is from 2010 to 2025.

The architecture of the model is implemented using PyTorch. The underlying structure of the PINN is a residual neural network, which is a feedforward network with skip connections between intermediate layers. It is implemented as a multi-layer perceptron with hyperbolic tangent as the nonlinear activation function. The physics component of the neural network comes from the physics-informed loss function. This function uses the model's outputs for value to calculate the FK equation as the residuals. The sum of the square of residuals is then used as a weighted component of the total loss function. The other component of the loss function is the mean squared error (MSE) from comparing the model's outputs for futures prices to the actual futures prices. As with most neural networks, the model will train by iterating through epochs that aim to minimize the loss function. To prevent overfitting, the model is cross validated using a random split of 80 to 20 percent of the data as training and testing, respectively.

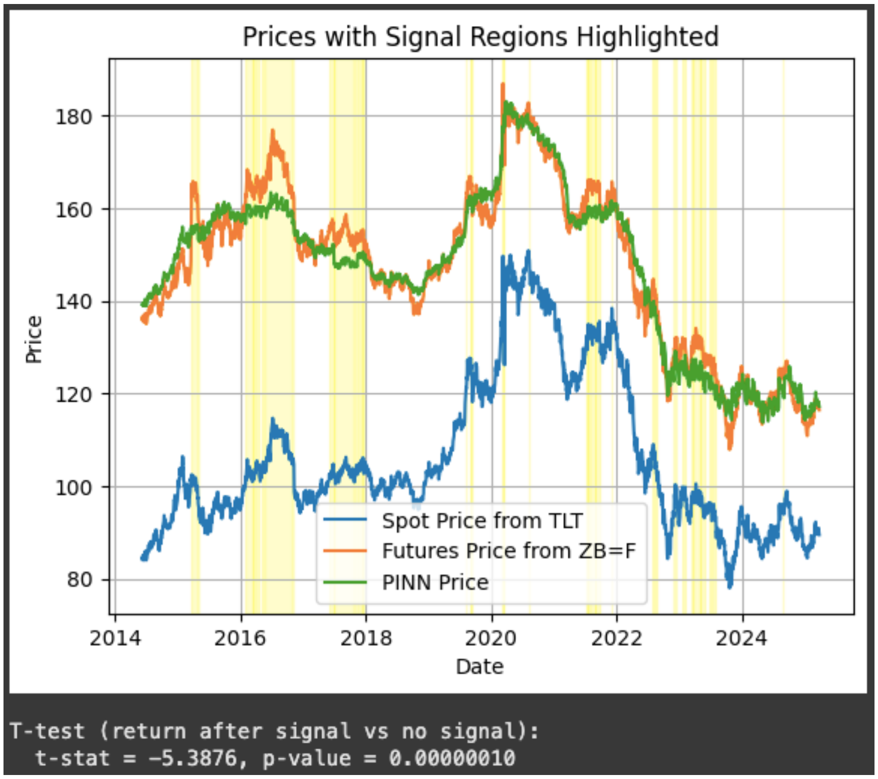
4 Results

After training the PINN across asset classes, I have found that it is effective at quickly calculating futures prices with minimal error. For each asset, the model was trained on 15 years of daily data using 10 parameters and 250 epochs, yet it took less than 1 minute to learn the pricing model. After training, the model is able to calculate prices for any range of input data almost instantaneously. Below is an example of the model training for 10-Year T-Note futures:

```
Epoch 150/250
Train Physics Loss: 0.0036, Train MSE Loss: 0.0018, Train Total Loss: 0.0029
Epoch 160/250
Train Physics Loss: 0.0036, Train MSE Loss: 0.0017, Train Total Loss: 0.0028
Epoch 170/250
Train Physics Loss: 0.0035, Train MSE Loss: 0.0017, Train Total Loss: 0.0028
Epoch 180/250
Train Physics Loss: 0.0034, Train MSE Loss: 0.0017, Train Total Loss: 0.0027
Epoch 190/250
Train Physics Loss: 0.0033, Train MSE Loss: 0.0017, Train Total Loss: 0.0027
Epoch 200/250
Train Physics Loss: 0.0032, Train MSE Loss: 0.0017, Train Total Loss: 0.0027
Epoch 210/250
Train Physics Loss: 0.0031, Train MSE Loss: 0.0017, Train Total Loss: 0.0026
Epoch 220/250
Train Physics Loss: 0.0030, Train MSE Loss: 0.0017, Train Total Loss: 0.0026
Epoch 230/250
Train Physics Loss: 0.0029, Train MSE Loss: 0.0017, Train Total Loss: 0.0025
Epoch 240/250
Train Physics Loss: 0.0028, Train MSE Loss: 0.0017, Train Total Loss: 0.0025
Epoch 250/250
Train Physics Loss: 0.0027, Train MSE Loss: 0.0017, Train Total Loss: 0.0025
--------------------------------------------------------------------------------
Final Test Physics Loss: 0.0026
Final Test MSE Loss: 0.0017
Final Test Total Loss: 0.0025
```

During training, the model effectively minimized loss by minimizing the model's MSE and physics loss, simultaneously. However, I noticed that it was quite common for MSE to plateau as the model continues minimizing the physics loss. Additionally, the physics, MSE, and total losses produced by the test sample were very similar to those of the training data. This means that the model did not overfit the training data, and it can successfully extrapolate the pricing model for new data. Across all assets tested, the lowest total loss on the testing data was 0.0007 from Dow Jones futures, and the highest total loss was 0.0040 from 30-Year T-Bond futures. Overall, the losses were very small, meaning our model was able to successfully create a pricing model for the futures contracts.

Now that our model performance has been validated, we can further examine the discrepancies between the accepted futures prices and the prices produced by the model. When I graphed the PINN prices against broker futures prices, I noticed a pattern where futures prices tended to drop dramatically after periods with a high spread between PINN and broker prices, particularly in the fixed income market segment. As a result, I generated signals for when the spread between the PINN price and broker price was a given number of standard deviations away from the mean spread. Then, I conducted a T-test to evaluate if there was a statistically significant relationship between the signals and a price drop in the futures contract. Below are my results for 30-Year T-Bond Futures using a standard deviation of 1 and a look-ahead period of 5 days:



Evidently, the signals produced statistically significant results, given that the p-value is 0.00000010. This indicates that bond futures may be overvalued at times, meaning a future price drop is probable.

5 Discussion

The PINN was able to successfully minimize the physics-informed loss function to calculate futures prices similar to those from the broker, while capturing supplemental information about the asset's volatility. The spread between the prices produced from the PINN and the actual futures prices can be used to generate signals that identify situations in which the futures price is likely to fall.

The PINN performed exceptionally well across asset classes, but insightful price discrepancies were most evident in fixed income futures. This being said, future research can be used to find optimal weights for the physics loss that can identify overvaluations in the futures contracts across all markets. Additionally, future research can involve setting a threshold to ensure that the model trains until it reaches a target loss rather than based on a stagnant number of epochs. Finally, the signals produced by the model should be backtested on historical data to quantify their performance for generating returns.

6 Conclusion

After conducting this research project, I have found that physics-informed neural networks are capable of successfully solving neural PDEs for calculating futures prices across asset classes. PINNs provide an efficient alternative to pure numerical methods, with a more wholistic perspective of the contract’s value. Pricings from PINNs can be used as a point of comparison to understand if the market is overvalued, and subsequently if the price will fall. The statistically significant relationship between the pricings spread and price drops has strong potential for alpha-generation.

7 References

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8 Appendices

Heston model SDE

$$dS_t = rS_t dt + S_t\sqrt{v_t}\Sigma_S^{\mathrm{T}} dW_t$$
$$dv_t = \kappa(\gamma - v_t)dt + \sigma\sqrt{v_t}\Sigma_v^{\mathrm{T}} dW_t$$

Feynman-Kac PDE

$$0 = \partial_t V - rV + \mu_y^{\mathrm{T}} \nabla_y V + \frac{1}{2} trace\left(\Sigma_y \Sigma_y^{\mathrm{T}} H_y(V)\right)$$

$$0 = \frac{\partial V}{\partial t} - rV + (\mu_S \frac{\partial V}{\partial S} + \mu_v \frac{\partial V}{\partial v}) + \frac{v}{2}(S^2 \frac{\partial^2 V}{\partial S^2} + 2\rho\sigma S \frac{\partial^2 V}{\partial v \partial S} + \sigma^2 \frac{\partial^2 V}{\partial v^2})$$

Terminal Condition

$$V(T, y_t) = S_t$$

Variables
V – value
t – time
T – maturity
r – risk-free rate
S_t – spot price (S)
v_t – variance (v)
μ_S – drift for spot price
μ_v – drift for variance
κ – rate of mean reversion
γ – long-run variance
ρ – correlation between Brownian motions

σ – volatility
Σ_y – diffusion matrix

Matrix Operations

$$y = \begin{bmatrix} S \\ v \end{bmatrix}$$

$$\mu_y = \begin{bmatrix} \mu_S \\ \mu_v \end{bmatrix} = \begin{bmatrix} rS \\ \kappa(\gamma - v) \end{bmatrix}$$

$$\mu_y \nabla_y V = \mu_S \frac{\partial V}{\partial S} + \mu_v \frac{\partial V}{\partial v}$$

$$\nabla_y V = \begin{bmatrix} \frac{\partial V}{\partial S} \\ \frac{\partial V}{\partial v} \end{bmatrix}$$

$$\Sigma_y = \sqrt{v} \begin{bmatrix} S & 0 \\ \rho\sigma & \sigma\sqrt{1 - \rho^2} \end{bmatrix}$$

$$\Sigma_y \Sigma_y^{\mathrm{T}} = v \begin{bmatrix} S^2 & \rho\sigma \mathrm{S} \\ \rho\sigma \mathrm{S} & \sigma^2 \end{bmatrix}$$

$$\mathrm{H}_y(V) = \begin{bmatrix} \frac{\partial^2 V}{\partial S^2} & \frac{\partial^2 V}{\partial S \partial v} \\ \frac{\partial^2 V}{\partial v \partial S} & \frac{\partial^2 V}{\partial v^2} \end{bmatrix}$$

$$trace\left(\Sigma_y \Sigma_y^{\mathrm{T}} H_y(V)\right) = v(S^2 \frac{\partial^2 V}{\partial S^2} + 2\rho\sigma \mathrm{S} \frac{\partial^2 V}{\partial v \partial S} + \sigma^2 \frac{\partial^2 V}{\partial v^2})$$

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